Higher Order Mixed Finite Elements for Maxwell's Equations

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Computational electromagnetics entails numerical solution of Maxwell's equations and has been one of the foundational pillars of modern electrical engineering. In this work, we demonstrate higher order, structure preserving finite element methods for the following system of Maxwell's equations:

$$\begin{aligned} \frac{\partial p}{\partial t} + \nabla E &= f_p \text{ in } \Omega \times (0,T], \\ \nabla p + \frac{\partial E}{\partial t} - \nabla \times H &= f_E \text{ in } \Omega \times (0,T], \\ \frac{\partial H}{\partial t} + \nabla \times E &= f_H \text{ in } \Omega \times (0,T], \end{aligned}$$

where $\Omega \subset \mathbb{R}^2/\mathbb{R}^3$ is a domain with Lipschitz boundary $\partial\Omega$ and with the following homogeneous boundary conditions: $p = 0, E \times n = 0, H \cdot n = 0$ on $\partial\Omega \times (0,T]$, where n is the unit outward normal to $\partial\Omega$, and with the following initial conditions: $p(x,0) = p_0(x), E(x,0) = E_0(x),$ and $H(x,0) = H_0(x)$ for $x \in \Omega$. We shall characterize the solution of this problem posed using a mixed variational formulation as follows.

Theorem 1 (Well Posedness)**.** *Let $f_p \in L^1[0,T] \times L^2(\Omega)$, $f_E \in L^1[0,T] \times L^2(\Omega)$, and $f_H \in L^1[0,T] \times L^2(\Omega)$. Then the solution (p, E, H) of the Maxwell's equations posed using a mixed variational formulation with the given initial and boundary conditions and with sufficient regularity satisfies:*

$$\|p\| + \|E\| + \|H\| \leq C\Big[\|p_0\| + \|E_0\| + \|H_0\| \ + \|f_p\| + \|f_E\| + \|f_H\|\Big],$$

for a positive bounded constant C and appropriate choices of the norms.

We shall then demonstrate computational results for some model problems in two and three dimensions using backward Euler and Crank-Nicholson schemes for the time discretization and finite elements for the spatial discretization. Our finite elements spaces shall be drawn from a de Rham sequence of conforming finite dimensional polynomial function spaces spanned by linear and quadratic Lagrange polynomial, and Nédélec and Raviart-Thomas vector basis elements.