Indraprastha Institute of Information Technology Delhi

Department of Mathematics

Syllabus for PhD Admission

Candidates are required to prepare topics from at least four of the following six general areas related to their area of interest.

1.1 Linear Algebra

System of Linear Equations, Matrices and Elementary Row Operations, Row-Reduced Echelon Matrices. Vector Spaces, Subspaces, Bases and Dimension, Ordered Basis and Coordinates. Linear Transformations, Rank-Nullity Theorem, The Algebra of Linear Transformations, Isomorphism, Matrix Representation of Linear Transformations, Linear Functionals, Annihilator, Double Dual, Transpose of a Linear Transformation. Characteristic Values and Characteristic Vectors of Linear Transformations, Diagonalizability, Minimal Polynomial of a Linear Transformation, Cayley-Hamilton Theorem, Invariant Subspaces, Direct-Sum Decompositions, Invariant Direct Sums, The Primary Decomposition Theorem, Cyclic Subspaces and Annihilators, Cyclic Decomposition, Rational and Jordan Forms. Inner Product Spaces, Orthonormal Basis, Gram-Schmidt Theorem.

1.2 Algebra

Group: Definition of groups, examples of groups, subgroups, cyclic groups, and generation of groups. Lagrange's Theorem and its consequences. Cosets and index of a subgroup. Normal subgroups and quotient (factor) groups. Group homomorphisms and isomorphisms, kernel and image of a homomorphism, and the First Isomorphism Theorem. Fundamental homomorphism theorems. Permutation groups and symmetric groups, alternating groups, cycle notation and cycle decomposition, properties of permutations. Group actions, orbits and stabilizers, class equation and applications. Structure of finite groups. Conjugacy classes, centralizers and normalizers. Sylow theorems and their applications. Direct products of groups. Finitely generated abelian groups and the classification theorem for finitely generated abelian groups.

Rings: Definition of rings, examples of rings and subrings. Ring homomorphisms and isomorphisms. Ideals and operations on ideals. Prime ideals and maximal ideals. Quotient rings and the isomorphism theorems for rings. Integral domains, units and zero divisors. Principal ideal domains (PIDs), Euclidean domains and unique factorization domains (UFDs). Divisibility in integral domains, greatest common divisors and the Euclidean algorithm. Polynomial rings over integral domains. Division algorithm for polynomials. Factorization of polynomials and irreducibility. Irreducibility tests including Eisenstein's criterion and reduction modulo p (mod-p irreducibility test). Gauss's lemma and primitive polynomials.

Fields: Fields and field extensions. Algebraic and transcendental extensions. Degree of an extension and simple extensions. Finite fields and their basic properties. Construction and structure of finite fields.

1.3 Real Analysis

Real Number System and its Completeness, Sequences and Series of Real Numbers. Metric Spaces: Basic Concepts, Continuous Functions, Completeness, Contraction Mapping Theorem, Connectedness, Intermediate Value Theorem, Compactness, Heine-Borel Theorem. Differentiation, Taylor's Theorem, Riemann Integral, Improper Integrals. Sequences and Series of Functions, Uniform Convergence, Power Series, Fourier Series, Weierstrass Approximation Theorem, Equicontinuity, Arzelà–Ascoli Theorem.

1.4 Complex Analysis

Topology of the Complex Plane, Riemann Sphere, Limits, Continuity and Differentiability, Analytic Functions, Harmonic Functions and Multivalued Functions. Convergence of Numerical Series, Radius of Convergence of Power Series and Power Series as an Analytic Function, Laurent Series. Cauchy's Integral Theorem, Cauchy Integral Formula, Morera's Theorem, Taylor's Theorem, Laurent's Theorem, Liouville's Theorem, Schwarz Lemma, Maximum Modulus Principle. Conformal Mappings, Linear Fractional Transformations, Classification of Singularities, Cauchy's Residue Theory and Evaluation of Real Integrals.

1.5 Functional Analysis

Normed Linear Spaces, Banach Spaces and Basic Properties: Heine-Borel Theorem, Riesz Lemma and Best Approximation Property. Inner Product Space and Projection Theorem, Orthonormal Bases, Bessel Inequality and Parseval's Formula, Riesz-Fischer Theorem. Bounded Operators and Basic Properties, Space of Bounded Operators and Dual Space, Riesz Representation Theorem, Adjoint of Operators on a Hilbert Space, Examples of Unbounded Operators, Convergence of Sequence of Operators. Hahn-Banach Extension Theorem, Uniform Boundedness Principle, Closed Graph Theorem and Open Mapping Theorem, Some Applications. Invertibility of Operators, Spectrum of an Operator.

1.6 Differential Equations

Ordinary Differential Equations:
Existence-Uniqueness: Review of Exact Equations of First Order, Method of Successive Approximations, Lipschitz Condition, Convergence of Successive Approximations. Existence and Uniqueness of Solutions for First Order Initial Value Problems, Non-Local Existence of Solutions, Existence and Uniqueness of Solutions to Systems, Existence and Uniqueness for Linear Systems, Equations of Order n. Second Order Equations: General Solution of Homogeneous Equations, Non-Homogeneous Equations, Wronskian, Method of Variation of Parameters. Sturm Comparison Theorem, Sturm Separation Theorem, Boundary Value Problems, Green's Functions, Sturm-Liouville Problems. Series Solutions of Second Order Linear Equations: Ordinary Points, Regular Singular Points,

Legendre Polynomials and Properties, Bessel Functions and Properties. Systems of Differential Equations: Algebraic Properties of Solutions of Linear Systems, Eigenvalue-Eigenvector Method, Complex Eigenvalues, Equal Eigenvalues, Fundamental Matrix Solutions, Matrix Exponential, Nonhomogeneous Equations, Variation of Parameters.

Partial Differential Equations:
Linear PDEs, First Linear and Quasilinear PDEs, Classification of Second Order PDEs, Cauchy Problem, Variable Separable. Wave Equation, Heat Equation, Laplace Equation, Transport Equation, D'Alembert's Principle, Boundary Value Problems, Green's Function.

1.7 Numerical Analysis

Norms of Vectors and Matrices. Linear Systems: Direct and Iterative Schemes, Ill-Conditioning and Convergence Analysis. Numerical Schemes for Nonlinear Systems, Regression. Numerical Solution of Differential Equations: Single Step and Multi-Step Methods, Order, Consistency, Stability and Convergence Analysis, Stiff Equations. Two Point Boundary Value Problems, Shooting and Finite Difference Methods.

1.8 Elementary Number Theory

Integers and equivalence relations, Divisibility: basic definition, properties, prime numbers, some results on distribution of primes; Congruences: basic definitions and properties, complete and reduced residue systems, theorems of Fermat, Euler and Wilson, linear congruences and Chinese Remainder theorem.

1.9 Probability

Sample space and events, conditional probability and Bayes theorem. Common univariate and multivariate distributions. Transformations of variables. Moment generating functions, probability generating functions, Markov and Chebyshev's inequalities, characteristic functions, modes of convergence, Borel-Cantelli lemmas, weak and strong laws of large numbers, central limit theorem. Basics of stochastic processes: Markov chains, branching process, random walks.

1.10 Statistics

Sufficiency, MVUE, maximum likelihood and other common methods of estimation. Tests for simple and composite hypotheses, Likelihood ratio and large sample tests, p-value, Confidence intervals/sets.

Standard sampling distributions. Basic properties of multivariate normal distribution, Wishart distribution, Hotelling's T2 and related tests. Analysis of discrete data – contingency, chi-square.

Simple random sampling, Systematic sampling, PPS sampling, Stratified sampling. Ratio and regression methods of estimation. Non-sampling errors, Non-response bias.

Partial and multiple correlations. Linear regression analysis and logistic regression.

1.11 Metric Spaces and Point-Set Topology

Metric spaces: Definition and examples of metric spaces. Open and closed balls, open and closed sets. Interior, closure, boundary, and limit points. Dense sets and separable spaces. Sequences and convergence in metric spaces. Cauchy sequences and completeness. Baire Category Theorem. Completion of a metric space. Continuous maps between metric spaces: ε-δ definition, sequential continuity, homeomorphisms. Uniform continuity. Isometries. Compactness: sequential compactness, total boundedness, equivalence in metric spaces. Connectedness and path-connectedness in metric spaces.

Topological spaces: Definition and examples (discrete, indiscrete, cofinite, metric topology, subspace topology). Bases and sub-bases for a topology. Open and closed sets, interior, closure, boundary. Continuous maps, homeomorphisms, and topological invariants. Compactness: open cover definition, finite intersection property. Connectedness and path-connectedness. Product topology and box topology. Quotient topology and quotient spaces.